# INVESTIGATION OF REVERSE FLOWS IN THE REGION OF SEPARATION OF THE TURBULENT BOUNDARY LAYER

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The mechanism of free turbulence is utilized for the solution of the problem of flows in the region of separating turbulent boundary layer.

The presence of reversed flows in the retarded zones is found. Cases of supersonic flow over a step (two-dimensional) and around a blunt body with a sting attached to the nose (axisymmetrical case) are examined.

The interaction of strong pressure gradients with turbulent boundary layer may cause separation, which is observed, for instance, on stings attached to the nose of blunt bodies [1] or in a flow over a step [2]. From experiments conducted in [2] it follows that the ratio of peak pressure rise in the stagnation zone ahead of the step to the free stream pressure does not change with increase in height of the step if it is more than twice the thickness of the boundary layer. Simultaneously with increase in height of the step, the point of separation moves upstream so that the ratio of step height to the distance from the step to the point of separation remains approximately constant for a given free stream $M$. The total pressure distribution in various cross-sections of the separated zone indicates presence of reversed flows in the retarded zone near the wall [2], while pressure gradients are found to be negligibly weak. These facts, and the fact of intensive turbulent mixing, observed in the retarded zone, suggests its investigation within the framework of free turbulence theory. In the light of this a retarded zone can be examined as a turbulent mixing zone adjacent to a semi-infinite free stream region.

Let $h$ be the height of a step in a supersonic flow and $\delta$ the thickness of the boundary layer. Assume that $h/\delta \gg 1$. In this case, separation begins near the point of intersection of the attached shock wave with the boundary layer. A rectilinear system of coordinates $x$, $y$ is selected so that the origin is at the point of separation and the $x$-axis is parallel to the velocity vector $U_0$ behind the attached shock wave. On Fig. 1, $a$ is the attached shock wave, $\partial$ the zone of reversed flows, $\varepsilon$ the boundary of the retarded flow. Let us limit ourselves to the examination of incompressible flow; compressible flow will be examined later. The equations of motion, continuity and energy in the case of two-dimensional free flow are

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial \tau}{\partial y}, \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$\rho c_p\left(u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial y}\right)=\frac{\partial q^*}{\partial y} \qquad \left(T^*=T+\frac{u^2}{2Ic_p}\right)$$

Here $\rho$ is the density, $T^*$ the temperature in the zone of retardation, $\tau$ the turbulent friction, $q^*$ the function which characterizes turbulent heat flow, $c_p$ the specific heat, $u$, $v$ the components of velocity along the axes.

The relationship between $\tau$ and $q^*$ and the mean flow is selected according to Taylor's vorticity transfer hypothesis

$$\frac{\partial \tau}{\partial y}=2\rho l^2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}, \qquad q^*=2\rho c_p l^2\frac{\partial u}{\partial y}\frac{\partial T^*}{\partial y} \qquad (l=cx)$$

Here, $l$ is the mixing path in Prandtl's theory. The constant $c$ characterizes the structure of the turbulent flow and is determined experimentally. As in Tollmien's problem, the velocity fields and temperature fields are assumed to be similar and dependent only upon the coordinate

$$\varphi = y/ax \qquad \left(a=\sqrt[3]{2c^2}\right)$$

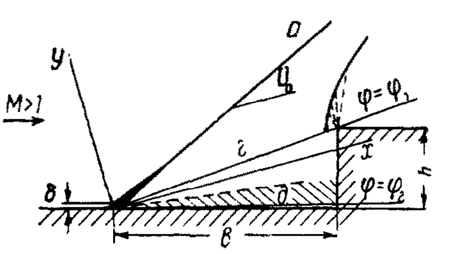


FIG. 1.

The zone of turbulent mixing will be limited by the straight rays, $\phi = \phi_1$ the interior boundary and $\phi = \phi_2$ the exterior boundary of the jet, the direction of which is determined by solution of the boundary-condition problem. Conditions on the exterior boundary must account for the presence of the reverse flows, and the inner boundary itself must be at a certain distance from the wall. Let us assume that conditions upon the ray $\phi_2$ are the same as on the exterior boundary, conditioned in turn by the reversed flow.

That assumption is equivalent to the statement that the ray $\phi = \phi_2$ coincides with the wall and is assumed to be a stream line.

The boundary condition will be

$$u = U_0, \quad v = 0, \quad \frac{\partial u}{\partial y} = 0 \qquad \text{at } \varphi = \varphi_1$$
$$\frac{u}{v} = \frac{x}{y}, \quad \frac{\partial u}{\partial y} = 0 \qquad \text{at } \varphi = \varphi_2$$

From the limiting conditions of shear stress at the point of separation, it follows that the shear stress along the ray $\phi = \phi_2$ must be equal to zero. From the general solution of Tollmien's equation

$$F''' + F = 0 \qquad \left(F' = \frac{dF}{d\varphi}, \; F(\varphi) = \int \frac{u}{U_0} d\varphi\right)$$

and satisfying the boundary condition, we find

$$\varphi_1 = 1.003, \quad \varphi_2 = -3.23, \quad u(\varphi_2) = -0.207\,U_0, \quad v(\varphi_2) = 0.67\,aU_0$$

Experimental values of the constant $a$, according to the various sources, are

$$a = 0.058\,(\text{Pabst}), \quad a = 0.0845\,(\text{Tollmien}), \quad a = 0.074\,(\text{Reichart})$$

Taking $a = 0.074$, we obtain the angle of the retarded zone equal to 17° 40′. The experimental value of the angle, according to [1], is approximately the same.

Knowing the angle of the retarded zone and the direction of flow on the interior ray $\phi = \phi_1$, the angle of the attached pressure gradient at a given free stream $M$ is found. The energy equation, as is well known, leads to a linear distribution of stagnation temperatures

$$\frac{T^* - T_2^*}{T_0^* - T_2^*} = \frac{\varphi - \varphi_2}{\varphi_1 - \varphi_2}$$

Here $T_0^*$ is the free-stream stagnation temperature

$T_2^*$ is the stagnation temperature at $\phi = \phi_2$.

In the case of the sting attached to the nose of a blunt-shaped body, it is necessary to note that the retarded zone has the shape of a cone and the stream lines are also conical.

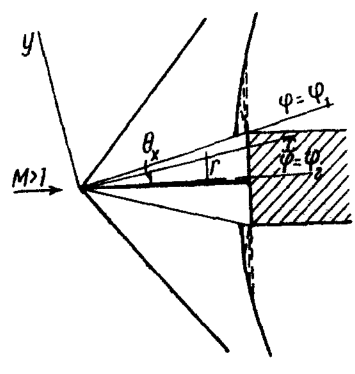


FIG. 2.

Let us examine the case when the retarded zone begins at the tip of the sting, which is selected as the origin of coordinate axes, $x$, $y$.

The direction of the $x$-axis with respect to the axis of the sting is determined by solving the problem of the free-stream boundary when the ray $\phi = \phi_1$ coincides with the conical boundary of the retarded zone. If $r(x, y)$ is the distance between the sting axis and the point which is being investigated (Fig. 2), then the equations of motion and continuity in the selected system of coordinates are

$$\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \frac{1}{r} \frac{\partial}{\partial y} (\tau r), \qquad \frac{\partial}{\partial x} (ur) + \frac{\partial}{\partial y} (vr) = 0$$

We now introduce the stream function

$$\psi = ax^2 U_x F(\varphi)$$

where $U_x$ is the projection of the stream velocity on the boundary of the retarded zone. Then the equation of motion is

$$-2F\left(\frac{F'}{\eta}\right)' = \left[\left(\frac{F'}{\eta}\right)^2 \eta\right]' \qquad \left(\eta(\varphi) = \frac{r}{x}\right)$$

Solving this equation for the boundary conditions which account for the presence of turbulent boundary layer on the sting, we find that the half-angle of the retarded zone is equal to 15.5°. This is close to the experimental value ($\sim$ 16°) taken from [1].

## BIBLIOGRAPHY

1. Mair, W.A., Experiments on separation of boundary layers on probes in front of blunt-nosed bodies in a supersonic air stream. *Phil. Mag.* No. 342, 1952.

2. Bogdonoff, S.M. and Kepler, C.E., Separation of a supersonic turbulent boundary layer. *JAS* No. 6, 1955.

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